Solving Quadratic Equations

A synthesis

1. Put your equation in Standard Form: $Ax^2 + Bx + C = 0$

2. If your equation has 2 terms:
   a) $Ax^2 + Bx = 0$. Solve by factoring. ex) $4x^2 - 3x = 0$
   $$x(4x - 3) = 0$$
   $$x = 0 \text{ or } x = \tfrac{3}{4}$$

   b) $Ax^2 + C = 0$. Solve by the square root method. ex) $4x^2 - 3 = 0$
   $$4x^2 = 3$$
   $$x^2 = \tfrac{3}{4}$$
   $$x = \pm \tfrac{\sqrt{3}}{2}$$

3. If your equation has 3 terms:
   a) Check to see whether the left side is factorable. If it is, solve the equation by factoring
   ex) $4x^2 - 8x + 3 = 0$ $AC = 12$, so -6 and -2 work
   $$4x^2 - 6x - 2x + 3 = 0$$
   $$2x(2x - 3) - 1(2x - 3) = 0$$
   $$(2x - 3)(2x - 1) = 0$$
   $$2x - 3 = 0 \text{ or } 2x - 1 = 0$$
   $$x = \tfrac{3}{2} \text{ or } x = \tfrac{1}{2}$$

   b) If your equation is not factorable, $A = 1$ and B is even; solve by completing the square.
   ex) $x^2 - 6x + 2 = 0$
   $$x^2 - 6x + \mathbf{9} = -2 + \mathbf{9} \qquad \left(\tfrac{b}{2}\right)^2 = \left(\tfrac{-6}{2}\right)^2 = 9 \quad \text{so add 9 to both sides}$$
   $$(x - 3)^2 = 7$$
   $$x - 3 = \pm\sqrt{7}$$
   $$x = 3 \pm \sqrt{7}$$

   c) If your equation is not factorable and A $\neq 1$, use the quadratic formula. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
   ex) $2x^2 + 3 = 2x$
   $$2x^2 - 2x + 3 = 0 \quad A = 2,\ B = -2,\ C = 3$$
   $$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(2)(3)}}{2(2)}$$
   $$x = \frac{2 \pm \sqrt{4 - 24}}{4}$$
   $$x = \frac{2 \pm \sqrt{-20}}{4}$$
   $$x = \frac{2 \pm 2i\sqrt{5}}{4} \text{ so } x = \frac{2(1 \pm i\sqrt{5})}{4} \text{ so } x = \frac{1 \pm i\sqrt{5}}{2}$$